Modern Physics Letters A, Vol. 3, No. 16 (1988) 1527–1532

AN INTERPRETATION OF COSMOLOGICAL MODEL WITH VARIABLE LIGHT VELOCITY

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Received 1 June 1988

A cosmological model with variable c, h, G is proposed. The characteristic lengths of physics (Compton, Jeans, Schwarzschild) are assumed to vary like $R(t)$. Both light and matter's worlds are found to obey the same law $R \approx t^{2/3}$. The Planck constant is found to vary like t and the gravitation one like $1/R$, while the Planck length vary like R. The particle masses follow $m \approx R$. The Hubble law still applies. The redshifts come from the secular variation of the Planck constant.

1. Introduction

Since 1930, the constancy of several so-called constants of physics have been criticized by many authors.[1,2,3,4] Accurate laboratory measurements show that these values appear quite constant in today's space time field, which is very small with respect to the whole space time, although Van Flandern[6] claimed observa tional evidence of the variation of the gravity constant G. As far as we can see, the extension of the constancy of the light velocity, and other so-called "fundamental constants" over the overall cosmic scale is a still debatable hypothesis. The purpose of this paper is to examine some of the consequences of a model in which the "constants" (especially the light velocity) are assumed to vary with time.

2. The Possible Secular Variation of c

Milne[1] first tried to propose an attempt of this type. He suggested that the observed redshifts are due to some secular change of the Planck constant and not to the classical Doppler effect. If the energy of the travelling photon remains constant, the apparent decrease of the observed frequency would only be due to the linear increase of h with the cosmic time t. In addition, Milne[1] suggested a decrease in time of the gravity constant G.

Similarly, Hoyle[2] argued against the assumption of the constancy of the mass content of the universe. He also suggested a secular change of G and constant creation of matter. Dirac,[3,4] starting from a hypothesis on the variation in time of some large numbers built with characteristic physical quantities (like the ratio of

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the electromagnetic force to the gravitational force), got a variable G and a constant creation of matter. Later, Canuto and Hsieh,[8] Lodenquai[5] and Julg[7] explored some consequences of Dirac's initial idea. But, surprisingly, no one contested the absolute constancy of c.

In the field equation, the so-called Einstein constant χ is determined by identification to the Poisson equation, which gives

$$\chi = -\frac{8\pi G}{c^2}. \tag{1}$$

The quantity χ must be an absolute constant with respect to the four dimensions, for the field equation to be divergenceless. But once the identification mentioned above refers to a steady situation, it does not imply the absolute constancy of G and c. A world model could a priori be built, with variable G and c, with respect to the cosmic time (which will be defined later) if the ratio G/c^2 remains an absolute constant.

In the sequel of the paper, we are going to analyze the effects of a secular variation of the light velocity.

3. Suggesting Gauge Relations

The Robertson-Walker metric, based on the isotropy and homogeneity assumptions, leads to the following system

$$-\frac{8\pi G}{c^2}\rho = \frac{3k}{R^2} + \frac{3}{R^2}\left(\frac{dR}{dx^0}\right)^2, \tag{2}$$

$$-\frac{8\pi G}{c^2}\left(\frac{p}{c^2}\right) = -\frac{k}{R^2} - \frac{1}{R^2}\left(\frac{dR}{dx^0}\right)^2 - \frac{2}{R}\frac{d^2R}{dx^{0^2}} \tag{3}$$

In this system, k is the sign of the curvature, p the pressure and ρ the density of energy-matter. In the classical model, we define the cosmic time t from the chronological variable x^0, by $x^0 = ct$, where c is considered as an absolute constant. In addition, the wavelength of the photon varies like R.

Let us consider the less restrictive condition

$$dx^0 = c(t)\,dt \tag{4}$$

which represents an alternative interpretation of the chronological parameter x^0. We are now going to relate the main physical constants to R, considered as a gauge parameter

$$c \approx \frac{1}{\sqrt{R}} \qquad \text{or} \qquad Rc^2 \approx \text{constant}, \tag{5}$$

$$m \text{ (particle's mass)} \approx R\,, \tag{6}$$

$$h \approx R^{3/2}\,, \tag{7}$$

$$G \approx 1/R\,. \tag{8}$$

With reference to relation (1), notice that $G/c^2 = \text{constant}$. In addition, if V is the relative velocity of a given element, for example, the random velocity of a galaxy in a cluster, or the velocity of a free particle in a cloud, we assume that V follows the secular variation

$$V \approx R\,. \tag{9}$$

If we suppose that the number of particles is conserved, the matter density ρ obeys

$$\rho \approx 1/R^2\,. \tag{10}$$

As a consequence, we can express the cosmic evolution through a gauge process, i.e., the Compton wavelength, the De Broglie wavelength, the Schwarzschild length and the Jeans length are found to vary like R.

In addition, our model still considers $mc^2 = \text{constant}$ and

$$\frac{mc^2}{\sqrt{1-\dfrac{V^2}{c^2}}} \approx \text{constant}\,. \tag{11}$$

The classical model saved the masses, assumed to be constant, but not the total matter-energy, through the variation of the cosmic background energy. In our scenario, it is just the reverse: the energy-matter is constant in time, not the masses. In addition, one should notice that the quantity Gm^2/R, which can be considered as a characteristic gravitational energy, is conserved.

Since the energies are conserved in our model, the moment as defined as mV^i varies like $R^{1/2}$. It is only constant if we define them as $\rho u^i c$.

Finally the Planck length varies with time like $R(t)$, the Planck time varies like t and the gravitational forces like $1/R(t)$.

4. The Evolution Equation

Introducing Eq. (4) into the systems (2), (3), we obtain the following equations

$$\frac{2R''}{R} + \frac{2R'^2}{R^2} + \frac{kc^2}{R^2} = \chi p\,, \tag{12}$$

$$\frac{1}{R^2}(R'^2 + kc^2) = -\chi\frac{\rho c^2}{3}. \qquad (13)$$

The use of the following equation of state

$$p = \frac{\rho\beta^2 c^2}{3} \qquad \text{with } 0 < \beta \leq 1 \qquad (14)$$

leads to

$$\frac{2R''}{R} + \frac{R'^2}{R^2}(2 + \beta^2) + \frac{kc^2}{R^2}(1 + \beta^2) = 0. \qquad (15)$$

In the case where $R = at^m$, the parameter β disappears from Eq. (15). From Eq. (5), $Rc^2 = R_0 c_0^2$ is an absolute constant, R_0 and c_0 being the present values of the gauge parameter R and light velocity c. The only possible value for k is -1, which means that in our model the curvature is negative. Then the evolution law becomes

$$R = \sqrt[3]{\frac{9}{4} R_0 c_0^2}\, t^{\frac{2}{3}}. \qquad (16)$$

Here, contrary to the classical models, light and matter obey the same evolution law. Moreover,

$$R = \frac{3}{2} ct. \qquad (17)$$

If we know t_0, the age of the universe, and c_0, the present value for the light velocity, we could derive the present value of the gauge parameter of the universe $R_0 = (3/2)\, c_0 t_0$, with the use of

$$L(t) \equiv \int_0^t c(\tau)\, d\tau \equiv R(t). \qquad (18)$$

The consequence is that the horizon is found to be identical, at any time, to the gauge factor $R(t)$.

5. Gauge Invariance of some Fundamental Equations

Let us take first the Vlasov equation, referring to collision free fluids. $f(\mathbf{r}, \mathbf{V}, t)$ is the velocity distribution function, which depends on the position vector $\mathbf{r}$, the velocity vector **V** and the time t. Ψ is the gravitational, e.g., $m\partial\Psi/\partial\mathbf{r}$ is the force acting on the particle whose mass is m.

$$\frac{\partial f}{\partial t} + \mathbf{V}\cdot\frac{\partial f}{\partial \mathbf{r}} - \frac{\partial \Psi}{\partial \mathbf{r}}\cdot\frac{\partial f}{\partial \mathbf{V}} = 0\,. \tag{19}$$

Introduce non-dimensional variables, such that

$$t = t^*\,\tau\,;f = f^*\,\xi\,;\mathbf{V} = V^*\,\mathbf{w}\,;\mathbf{r} = R^*\,\zeta\,;\Psi = (G^*\,m^*/R^*)\,\varphi$$

Equation (19) becomes

$$\frac{1}{t^*}\frac{\partial \xi}{\partial \tau} + \frac{V^*}{R^*}\mathbf{w}.\frac{\partial \xi}{\partial \zeta} - \frac{G^*\,m^*}{R^{*2}\,V^*}\frac{\partial \varphi}{\partial \zeta}\cdot\frac{\partial \xi}{\partial \mathbf{w}} = 0\,. \tag{20}$$

Introduce the precedent gauge relations $G^* \approx 1/R^*$, $m^* \approx R^*$. The dimensional analysis of Eq. (20) gives $V^* \approx 1/(R^*)^{1/2}$ and

$$R^* \approx t^{*2/3}. \tag{21}$$

These relations can be interpreted as gauge relations and related to the solution (16). Consider now the Schrödinger equation

$$-\frac{h^2}{2m}\nabla^2\Psi + U\Psi = -i\,h\frac{\partial \Psi}{\partial t}\,. \tag{22}$$

Introduce $t = t^*\,\tau$, $\mathbf{r} = R^*\,\boldsymbol{\zeta}$, $h = h^*\eta$, $m = m^*\,\mu$, $U = U^*u$. The dimensional analysis of Eq. (22) gives

$$\frac{h^{*2}}{2m^*\,R^{*2}} \approx U^* \approx \frac{h^*}{t^*}, \qquad \text{i.e.,} \qquad \frac{t^{*2}}{R^{*3}} \approx U^* \approx \text{constant}, \tag{23}$$

i.e., $R^* \approx t^{*2/3}$. Now let us write the Maxwell equations, referring to an empty space

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}\,, \tag{24}$$

$$\nabla \times \mathbf{B} = \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}\,, \tag{25}$$

and write $\mathbf{E} = E^*\,\boldsymbol{\varepsilon}$, $B = B^*\boldsymbol{\beta}$, $\mathbf{r} = R^*\,\boldsymbol{\zeta}$, $t = t^*t$, $c = c^*\,\omega$. We get

$$\frac{E^*}{R^*}\frac{\partial}{\partial \zeta} \times \varepsilon = -\frac{B^*}{t^*}\frac{\partial \beta}{\partial \tau} \tag{26}$$

$$\frac{E^*}{R^*}\frac{\partial}{\partial \zeta} \times \beta = \frac{E^*}{c^{*2}\,t^*}\frac{\partial \varepsilon}{\partial \tau}. \tag{27}$$

Combining with $c^* \approx 1/R^{*1/2}$, we refind $R^* \approx t^{2/3}$.

6. Conclusion

In this paper, we have derived some of the implications of letting the fundamental constants vary with time. This can only be done with the addition of some further gauge constraints. Following Milne's suggestion[1], the classical interpretation of the redshift in terms of the Doppler effect has to be replaced by another one taking into account the secular change of the Planck constant. The fundamental parameters R and c are related to each other by some gauge relation. The particle masses vary like R while the energy-matter and the gravitation energy are conserved.

This model predicts that the cosmological horizon $L(t)$ should be identical to $R(t)$, which would justify the overall homogeneity of universe. The curvature of space should be negative and the gauge relationship between R and t should be $R \approx t^{2/3}$.

The Planck's constant would vary like t, and the gravitational constant G like $1/R$, the Planck's length would vary like R, and the Planck's time would vary like t. The gravitational force would vary like $1/R$.

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