Modern Physics Letters A, Vol. 3, No. 18 (1988) 1733–1744

COSMOLOGICAL MODEL WITH VARIABLE LIGHT VELOCITY: THE INTERPRETATION OF RED SHIFTS

JEAN-PIERRE PETIT
*Observatoire de Marseille**

Received 11 July 1988

The model with variable c, G, h presented in Ref. 1 is extended to electromagnetism. The entropy is found to vary like $\log t$ and, in a space-entropy representation, the metric is conformally flat. A new gauge relation is suggested, based on geometrical considerations, which corresponds to a Rydberg constant varying like R. The Hubble's law still applies. The age of the universe is unchanged while its span is found to be half of the Mattig's value. The complete decoding of the red shift can be done. The distances of the sources are very similar. The large volumic power densities of distant quasars could have been greatly overestimated, while the increase of their absolute magnitude, as derived from the classical theory, could be due to the secular variation of c. Assuming the electron-proton mass ratio to vary like R, we get a fine structure constant α, a Bohr radius and a ratio of electromagnetic force to gravitational force which behave like absolute constants.

1. Introduction

Several authors tried to develop models with physical constants in time varying with time.[2,3,4,5,6,7,8,9] None questioned the light velocity c, always considered an absolute constant. If one wants to save the form of the conservation equations, the Einstein's constant $\chi = -8\pi G/c^2$ must be considered as an absolute constant. In such conditions, if one wants to keep the light velocity c as an absolute constant c, and a variable gravitational constant G, one must add a source term to the field equation (see Ref. 3). Thus, all these theories imply a constant creation of matter.

In a previous paper[1] we showed that a variable light-velocity could lead to a consistent model if both G and the Planck's constant h followed convenient gauge relations. Thus, the constant creation of matter was no longer necessary. The extension of the Robertson-Walker metric to a variable light velocity configuration and its introduction into the field equation gave a complete set of gauge relations. Let us recall the main features of the model described in Eq. (1)

$$c \approx \frac{1}{\sqrt{R}} \approx t^{-1/3} \tag{1}$$

* Postal address: Chemin de la Montagnère, 84120 Pertuis, France.

$$m \text{ (particle's mass)} \approx R \tag{2}$$

$$h \text{ (Planck' constant)} \approx R^{3/2} \approx t \tag{3}$$

$$G \text{ (gravity constant)} \approx 1/R \tag{4}$$

$$R \text{ (characteristic length)} \approx t^{2/3} \tag{5}$$

$$V \text{ (velocity of a free particle)} \approx R^{-1/2} \approx c \tag{6}$$

$$\rho \text{ (mass density)} \approx 1/R^2 \tag{7}$$

$$mc^2 = \text{constant}. \tag{8}$$

The following is a short digression on entropy.

2. Time or Entropy?

The relativistic formulation of the velocity distribution function is

$$f = n\left(\frac{m}{2\pi kT}\right)^{3/2} \frac{1}{cK_2\left(\frac{mc^2}{kT}\right)} \sqrt{\frac{2\pi kT}{m}} \exp\left(-\frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}\right), \tag{9}$$

where m is the rest mass, T the temperature, n the number of density and K_2 a Bessel function. If $\beta = (\langle v^2 \rangle^{1/2}/c) \ll 1$, then we get the classical Maxwell-Boltzmann velocity distribution function

$$f = n\left(\frac{m}{2\pi kT}\right)^{3/2} e^{-mv^2/2kT}. \tag{10}$$

Let us compute the entropy per baryon, as defined by

$$s = -\frac{k}{n}\int\int\int f \log f \, du \, dv \, dw = -k\langle \log f \rangle \tag{11}$$

where k is the Boltzmann's constant. We have $n \approx R^{-3}$, $m \approx R$ and $Rt^{2/3}$, $T =$ constant (see Ref. 1), such that

$$\log f = \log A(\beta) - \log t - \frac{2}{\beta^2 \sqrt{1-\frac{v^2}{c^2}}}. \tag{12}$$

Then

$$s = k \log t + H(\beta). \tag{13}$$

In the model, β is gauge invariant such that $s \approx \log t$.

In the classical cosmology, the universe is isentropic. One could consider it somewhat paradoxical that such an enormous change in time goes with an almost zero entropy variation. In the new model, the entropy grows with time. Notice that the BIG BANG singularity corresponds to $s = -\infty$.

Let us now define the entropy through

$$s = 3/2k \log t. \tag{14}$$

Let us return to the Robertson-Walker metric

$$dS^2 = c^2\, dt^2 - R^2 \frac{du^2 + u^2\, d\theta^2 + u^2 \sin^2\theta\, d\varphi^2}{(1+u^2)^2}. \tag{15}$$

We get

$$dS^2 = R^2 \left\{ ds^2 - \frac{du^2 + u^2\, d\theta^2 + u^2 \sin^2\theta\, d\varphi^2}{(1-u^2)^2} \right\}. \tag{16}$$

In the representation {entropy, space variables}, the metric is *conformally flat*. From a cosmological point of view, the entropy (which is invariant with respect to the Lorentz transform) could be a better choice than time.

In addition, if we describe the universe in a phase space (position plus velocity), we find that the associated characteristic hypervolume R^3c^3 varies similarly to t.

3. The Red Shift and the Robertson-Walker Metric

Consider a radiating object, say a nebula N_1, which could be considered as a particle. Suppose his light is observed on a nebula N_2 placed at the origin of the co-moving coordinates. The nebula N_1 is characterized by the value of its time-independent distant marker ζ, defined by the relation

$$d\zeta = \frac{\{du^2 + u^2(d\theta^2 + \sin^2\theta\, d\varphi^2)\}^{1/2}}{1 + \frac{k}{4}u^2}. \tag{17}$$

The light emitted at time t_1 is observed on N_2 at a time t_2 with $t_2 > t_1$.

The distance between N_1 and N_2 is $R(t)\, d\zeta$ and is time-dependent, but $d\zeta$ is not. Light travels on a null geodesic

$$ds^2 = (dx^0)^2 - R^2\, d\zeta^2 = 0\,,$$
$$\zeta = \int_{x_1^0}^{x_2^0} \frac{dx^0}{R(x^0)} \text{ is an invariant.} \tag{18}$$

Consider the light emitted by N_1 at a corresponding value $x_1^0 + \Delta x_1^0$ of the chronological parameter. It will be received at $x_2^0 + \Delta x_2^0$, where it will be determined through the relation

$$\int_{x_1^0+\Delta x_1^0}^{x_2^0+\Delta x_2^0} \frac{dx^0}{R(x^0)} = \zeta\,. \tag{19}$$

Consider Δx_1^0 as the equivalent of the period of some physical phenomenon, the emission of radiation for instance, taking place on N_1 and Δx^0 to be short compared to the equivalent of the travel time from N_1 to N_2 (in terms of the chronological parameter x^0). The periodic phenomenon will appear, as seen from N_2, to have a "period" Δx_2^0 which, from the above relation, will be such that the increment of the θ integral will be zero.

This, by elementary calculus, gives

$$\frac{\Delta x_2^0}{R(x_2^0)} - \frac{\Delta x_1^0}{R(x_1^0)} = 0\,. \tag{20}$$

Let us introduce the cosmic time t as defined before through $dx^0 = c\,dt$ and write $R(x_1^0) = R_1$, $R(x_2^0) = R_2$, $c(x_2^0) = c_2$, $c(x_1^0) = c_1$, then we get

$$\frac{\Delta t_2}{\Delta t_1} = \frac{R_2}{R_1}\frac{c_1}{c_2}\,. \tag{21}$$

4. The Problem of Electromagnetism

Now we cannot assert that the energy $E_i(t_1)$, emitted by the atom at time t_1 would be identical to the corresponding emission energy $E_i(t_2)$ of a similar atom, at time t_2, in laboratory conditions. The light emission is an electromagnetic process. Everyone knows that the classical field description, applied to a four-dimensional space time does not take in charge the electromagnetic phenomenon. To get a complete description of the universe, gravitation and electromagnetism should be imbedded in a common geometrical framework. Unfortunately, it has not yet been done in a satisfactory way so that our work will now lose somewhat its self-consistency. Suppose, for instance, that the Rydberg constant (ionization energy of hydrogen) would obey simply the following hypothetic gauge relationship

$$E_i \approx R\gamma . \tag{22}$$

(Notice that this is a totally arbitrary assumption.) Let us explore the consequences of it on the red shift decoding. Later we will try to relate it to possible gauge relations.

5. The Red Shift Phenomenon

In the classical description, the red shift z is due to the Doppler effect, plus some special relativity additional effect. The index 1 refers to the emitter and the index 2 to the receiver. For a given spectroscopic line, call $E_1 = h_1 v_{10}$, the emission energy, and $E_2 = h_2 v_{20}$, the corresponding emission energy, in today's lab's conditions, for the same line. The light is emitted by an atom at rest at the frequency $v_1 = v_{10}$, corresponding to the wavelength $\lambda_1 = c_1/v_1 = \lambda_{10}$.

Here, v_2 will be the measured reception frequency, with $\lambda_2 = c_2/v_2$ and $\lambda_{20} = c_2/v_{20}$. The energy of any radiative process will follow the general assumed gauge law (22).

We can define the red shift z:

1. As the ratio between the wavelengths

$$1 + z = \frac{\lambda_2}{\lambda_{20}} = \frac{\lambda_2}{\lambda_1}\frac{\lambda_{10}}{\lambda_{20}}, \tag{23}$$

$$\begin{gathered} \frac{h_1 v_{10}}{h_2 v_{20}} \approx \left(\frac{R_1}{R_2}\right)^{\gamma} \text{ with } \frac{h_1}{h_2} \approx \frac{t_1}{t_2} \approx \left(\frac{R_1}{R_2}\right)^{3/2}, \\ \frac{v_{10}}{v_{20}} \approx \left(\frac{R_1}{R_2}\right)^{\gamma}\left(\frac{R_2}{R_1}\right)^{3/2} = \left(\frac{R_1}{R_2}\right)^{\gamma-(3/2)}, \\ \frac{\lambda_{10}}{\lambda_{20}} \approx \frac{c_1}{c_2}\frac{v_{20}}{v_{10}} \approx \left(\frac{R_2}{R_1}\right)^{1/2}\left(\frac{R_2}{R_1}\right)^{\gamma-(3/2)} = \left(\frac{R_2}{R_1}\right)^{\gamma-1}, \end{gathered} \tag{24}$$

then we get

$$1 + z = \left(\frac{R_2}{R_1}\right)^{\gamma}. \tag{25}$$

Notice that, for $\gamma = 1$, we refind the classical model.

2. As the ratio between the frequencies

$$1 + z = \frac{v_{20}}{v_2} \text{ with } v_2 = \frac{c_2}{\lambda_2} \text{ and } v_{20} = \frac{c_2}{\lambda_{20}}, \tag{26}$$

we get the same result.

3. As the ratio between the energies

$$1 + z = \frac{h_2 v_{20}}{h_1 v_{10}} \approx \left(\frac{R_2}{R_1}\right)^{\gamma}. \tag{27}$$

We obtain the same result. The classical relation suggests the choice of $\gamma = 1$.

6. The Hubble's Law and the Robertson Walker Metric

Let us expand the function $1/R(t)$ into a series with respect to

$$\varepsilon = \frac{c_2(t - t_2)}{R_2}, \tag{28}$$

we get

$$\frac{1}{R(t)} = \frac{1}{R_2} + \frac{R_2'}{R_2 c_2}\varepsilon + \frac{1}{c_2^2}\left(\frac{R_2'^2}{R_2} - \frac{R_2''}{2}\right)\varepsilon^2 + O(\varepsilon^3). \tag{29}$$

In R_2' and R_2'', the prime denotes differentiation with respect to t. In particular, at the first order,

$$z = \left(\frac{R_2}{R_1}\right)^{2-\gamma} - 1 = (2 - \gamma)\frac{R_2'}{c_2}\varepsilon. \tag{30}$$

Next, expanding the following expressions

$$\int_{t_1}^{t_2} \frac{c\,dt}{R} = \zeta,$$

$$\zeta = \frac{c_2}{R_2}(t_2 - t_1) + \left(\frac{c_2}{R_2}\right)' \frac{\{t_2 - t_1\}^2}{2} + O\{(t_2 - t_1)^3\} \tag{31}$$

$$\zeta = \varepsilon + \frac{1}{2c_2}\left(\frac{c_2}{R_2}\right)' \varepsilon^2 + O(\varepsilon^3).$$

Referring to the first order,

$$c_2 z \cong (2 - \gamma) R_2' \zeta. \tag{32}$$

As a first approximation, the astronomer measures $d_2 \cong R_2 \zeta$, such as

$$c_2 z \cong (2 - \gamma)\frac{R_2'}{R_2} d_2 \tag{33}$$

Which is nothing but the Hubble's red shift law, which still applies in these variable light velocity conditions. From the measurement of d_2, c_2 and z, we can derive the so-called Hubble's constant, i.e., the age of universe.

Take $R = 3/2ct$ (see Ref. 1).

$$R' = \frac{3}{2}(c + tc') = \frac{3}{2}c\left(1 + t\frac{d \log c}{dt}\right) = c. \tag{34}$$

The age of universe corresponds to

$$t = (2 - \gamma)\frac{2}{3}\frac{d_2}{c_2 z}. \tag{35}$$

A $\gamma = 1$ value gives the standard model value.

7. The Red Shift and the Distance Evaluation

Let us return to the Robertson-Walker metric, which provides

$$\int_{t_1}^{t_2} \frac{c\,dt}{R} = \int_0^u \frac{dw}{1 + k\frac{w^2}{4}}. \tag{36}$$

In the classical approach, take the Einstein-de Sitter model ($k = 0$). We get

$$\int_{t_1}^{t_2} \frac{c\,dt}{R} = u. \tag{37}$$

With $R = at^{2/3}$, we have

$$\int_{t_1}^{t_2} \frac{c\,dt}{R} = \frac{3c}{a}(t_2^{1/3} - t_1^{1/3}) = \frac{3c}{a}\left(1 - \frac{1}{\sqrt{1 + z}}\right) = \frac{3ct_2}{at_2^{2/3}} = \frac{3ct_2}{R_2} = u,$$

whence

$$d_2 = R_2 u = 3ct_2\left(1 - \frac{1}{\sqrt{1 + z}}\right). \tag{38}$$

If z is weak, $d_2 \approx 3/2ct_2 z$; if z tends to infinity, d_2 tends to $3ct_2$.

Of course the Mattig's formula gives the same result

$$u = \frac{c}{R_2 H_2 q_2^2 (1+z)} \{q_2 z + (q_2 - 1)\sqrt{1 + 2q_2 z} - 1\},$$

$$q_2 = \frac{1}{2} \to u = \frac{4c}{R_2 H_2 (1+z)} \left(\frac{z}{2} - \frac{1}{2}\sqrt{1+z} + \frac{1}{2}\right),$$

$$u = \frac{2c}{R_2 H_2}\left(1 - \frac{1}{\sqrt{1+z}}\right), \quad d_2 = R_2 u, \quad H_2 = \frac{2}{3t_2}.$$

Let us return to the Robertson-Walker metric, following our model, with $k = -1$

$$ds^2 = (dx^0)^2 - e^{g(x^0)} \frac{b^2}{1 - \frac{abr^2}{4}} (dr^2 + r^2 \, d\theta^2 + r^2 \sin^2\theta \, d\varphi^2),$$

we write

$$\frac{|ab|}{4} = \frac{1}{r_0^2} \quad \text{or} \quad b = \frac{4}{ar_0^2}.$$

$$ds^2 = (dx^0)^2 - \frac{e^{g(x^0)}\left(\frac{16}{a^2 r_0^4}\right)}{\left(1 - \left(\frac{r}{r_0}\right)^2\right)} (dr^2 + u^2 \, d\theta^2 + u^2 \sin^2\theta \, d\varphi^2).$$

Let

$$e^{g(x^0)} \frac{16}{a^2 r_0^4} = R(t).$$

Then

$$ds^2 = c^2 \, dt^2 - R^2 \frac{(du^2 + u^2 \, d\theta^2 + u^2 \sin^2\theta \, d\varphi^2)}{(1 - u^2)^2}.$$

For radial paths,

$$c\, dt = \frac{R\, du}{1 - u^2}, \quad \int_{t_1}^{t_2} \frac{c\, dt}{R} = \operatorname{Arg\,th} u,$$

$$c = c_2\left(\frac{t_2}{t}\right)^{1/3}, \quad R = R_2\left(\frac{t}{t_2}\right)^{2/3},$$

$$\log(1+z) = \operatorname{Arg\,th} u,$$

$$u = \frac{(1+z) - \dfrac{1}{(1+z)}}{(1+z) + \dfrac{1}{(1+z)}},$$

$$d_2 = R_2 u = \frac{3}{2}c_2 t_2 \frac{(1+z)^2 - 1}{(1+z)^2 + 1}. \tag{39}$$

When z tends to infinity, we refind the horizon $(3/2)c_2t_2$, which is twice smaller than the standard value $3c_2t_2$.

Notice that this is completely similar to the law giving v_r/c (where v_r is the radial velocity) as a function of z, in the standard model.

Let us compare the distances as given by our model and the standard model,

$$\eta = \frac{(1+z)^2 - 1}{(1+z)^2 + 1} \frac{1}{2 - \dfrac{2}{\sqrt{1+z}}}. \tag{40}$$

They are similar for weak z values.

8. The Quasars Problem

Quasars correspond presently to z values ranging from 0.13 to 4. The diameters of quasars are estimated from their fluctuation period T. We get a maximum diameter of cT. With respect to the standard approach, this model gives larger values, for c was larger in the earlier time.

The volumic power is referred to the size of the galaxies. Call P_{QSO} the absolute power emitted by a quasar and P_{G} the absolute power emitted by a galaxy. The relative power density of the QSO, with respect to a galaxy, is

$$\pi = \frac{P_{\mathrm{QSO}}}{(cT)^3} \frac{\text{Volume galaxy}}{P_{\mathrm{G}}}. \tag{41}$$

But in our model the galaxies are no longer constant in size. They grow with time. Suppose the quasar is imbedded in a galaxy. The size of this galaxy will grow like $(1+z)$. As such our correcting term, with respect to the standard values for power density, will involve three effects

a. change for the distance (they are a little bit closer); b. change for the diameter (due to the variation of c); c. change of the galaxy's size.

Given P_{QSO} and P_G, the coefficient (57) becomes $\pi' = \pi\xi$ with

$$\xi = \eta^2 \frac{1}{(1+z)^3} \frac{1}{(1+z)^{3/2}} = \frac{\eta^2}{(1+z)^{9/2}}, \tag{42}$$

i.e.,

$$\xi = \frac{1}{4}\left\{\frac{(1+z)^2 - 1}{(1+z)^2 + 1}\right\}^2 \frac{1}{(\sqrt{1+z} - 1)^2 (1+z)^{7/2}}. \tag{43}$$

z	η	ξ	
0	1	1	
0.13	1.025	0.606	the closest quasar
0.2	1.03	0.467	
0.5	1.048	0.177	
1	1.024	0.0463	
1.5	0.985	0.0157	
2	0.946	0.0063	
2.5	0.912	0.00296	
3	0.882	0.00152	
3.5	0.856	0.000842	
4	0.834	0.0005	the most distant quasar
5	0.8	0.000201	
6	0.771	0.0001	
7	0.75	0.0000485	
8	0.731	0.0000275	
9	0.71	0.0000162	
10	0.7	0.0000102	

We see that this correction reduces the absolute magnitude of the observed quasar, and that this correction increases with z. Thus, if this model is good, the classical model would have greatly overestimated the volumic power density of quasars. In addition, the observed increase of absolute magnitude of quasars could be due to the secular change in c. Classically, the galaxies' span is related to the Jeans' length, but the model does not provide any available information about the sizes of some emitting objects like stars or quasars. It depends on the energy emission process. As we have not defined a possible gauge relation for the fusion coefficients, we have no available model yet. Anyway the quasars could grow in time, like galaxies, and the observations tend to support this hypothesis. That will be examined in more detail in the next paper devoted to the detailed interaction of the model and available observations.

9. Associated Gauge Relations

The ionization energy of hydrogen obeys $E_i = 1/2\alpha^2 m_e c^2$, where α is the fine structure constant and m_e the mass of the electron. We have assumed that $E_i \approx R\gamma$ with $\gamma = 1$, in order to fit with the classical model. (See Eqs. (25), (33) and (35).) Introduce the electron-proton mass ratio $\delta = m_e/m_p$. According to the first paper,[1] $m_p \approx m_n \approx m \approx R$ such that mc^2 is an absolute constant. Then

$$\alpha^2\delta \approx R. \tag{44}$$

The fine structure constant α and the electron-proton mass ratio δ cannot be kept constant together. We shall consider two possibilities.

9.1. *Let us take first $\delta \approx$ constant*

Then

$$\alpha = \frac{e^2}{2\varepsilon_0 hc} \approx \sqrt{R}. \tag{45}$$

Introducing the gauge relations for h and c, we get: $e^2/\varepsilon_0 \approx R^{3/2}$ and the electromagnetic force $F_{em} = e^2/4\pi\varepsilon_0 R^2 \approx R^{-1/2}$. Then

$$\frac{\text{Gravitational force}}{\text{Electromagnetic force}} \approx \frac{1}{\sqrt{R}}, \tag{46}$$

which is similar to an old idea of Dirac (Refs. 4, 5).

Let us compute the Bohr radius

$$a_0 = \frac{h^2\varepsilon_0}{\pi m_e e^2} \approx \sqrt{R}. \tag{47}$$

9.2. *Now we take $\alpha \approx$ absolute constant*

Then $m_e \approx R^2$ and $e^2/\varepsilon_0 \approx R$. The Compton length of the electron $h/m_e c$, the ratio of gravitational force to the electromagnetic force and the Bohr radius become absolute constants.

In Refs. (10) to (13) several authors studied the possible variability of several quantities: α, $\alpha^2(g_p/g_e)(m_e/m_p)$, m_e/m_p, where g_p and g_e are the gyromagnetic ratios of the proton and the electron. Following B.E.G. Pagel,[13] we have

Effect	Quantity	Approximate 3σ upper limit to variation
Optical doublet splitting.	α	3%
Comparison of optical and 21 cm redshifts.	$\alpha^2(g_p/g_e)(m_e/m_p)$	10^{-3}
Comparison of hydrogen and metal redshifts.	$\delta = m_e/m_p$	50%

As a consequence, we choose the second possibility, with a variable mass ratio. Notice that in both cases we get $g_p/g_e \approx 1/R$.

10. Conclusion

Here we tried to extend the model introduced in Ref. 1 to electromagnetism. A gauge law was suggested: we assumed the ionisation energy E_i (Rydberg constant) to vary like $R\gamma$. Local geometrical considerations recommend the value $\gamma = 1$, which takes into account the desonization process during the cosmic evolution. The distance of a radiative source, as derived from the Robertson-Walker metric, gives results quite similar to the standard model values, but this new model tends to reduce considerably the estimated density power of distant sources like quasars. In addition, the increase of absolute magnitude in z, as derived from the classical model, could be due to the secular variation in c.

With an electron-proton mass ratio $\delta = m_e/m_p$ which varies like R, we get a fine structure constant α which behaves like an absolute constant.

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